出國報告（出國類別：發表學術論文）

資訊科學與應用數學的國際會議

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摘要

本文係針對海軍官校應用科學系陳冠如教授出國發表學術論文之過程進行報告。報告內容包括參加會議目的、參加會議過程、與會心得及建議事項。

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一、會議目的

World Academy of Science, Engineering and Technology is a scientific society of distinguished scholars engaged in scientific, engineering and technological research, dedicated to the furtherance of science, engineering and technology. 本會議為第三十二屆資訊科學與應用數學的國際會議。這會議讓計算機科學和應用數學的學術科學家、研究員和學者交換和分享他們的經驗和研究結果。WASET 會議每年在世界各地舉辦數個重要國際研討會，本人參加今年於 01 月 29 至 01 月 31 日為期三天在阿拉伯聯合大公國的杜拜舉行的會議，與會者約數百餘人，發表論文達數百篇；其目的在分享各國數學學者研究之成果。本人所發表論文題目：

中文題目：利用山路引理來證得漸進線性薛丁格方程式解的存在性

英文題目：Applying the mountain pass theorem for a class of asymptotically linear Schrodinger equation

二、參加會議經過

非常感謝國科會自然處的經費支援，使得本人得以參加 2012 年 01 月的第三十二屆資訊科學與應用數學的國際會議在阿拉伯聯合大公國的杜拜所辦的國際研討會，參加之各地的學者專家十分踴躍，將近有數百餘人註冊，大多為中東、歐美人士，大會選在阿拉伯聯合大公國的杜拜的一家飯店舉行。大會總共發表了百多場大會學術演講，及二百多個海報張貼論文。此次的會議內容主題主要是計算機科學在數學上的應用，讓會議的參與者除了可以和各國學者分享自己的研究成果以外，也有機會可以聆聽與學習各國學者的研究主題，充實自己在研究領域中的基本知能。

本人同時於會中發表海報張貼學術論文。所發表海報論文

中文題目：利用山路引理來證得漸進線性薛丁格方程式解的存在性

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此偏論文很高興已被 WASET 收錄到期刊。

三、與會心得

首先要感謝行政院國家科學委員會自然處給予本人出席國際會議的差旅補助，讓本人有機會去見識到所謂的國際大型會議，參與過程本人增加不少國際觀與吸收到專業的研究知識。此次參與會議和往年參加的國際會議有一個最大的不同，即是會議在飯店舉行，感覺有點不夠有學術氛圍，且閒雜人等太多，常有路人來探頭探腦參觀，影響演講者情緒。

現在正是杜拜的冬季，氣候宜人，大體而言，整個環境給人相當舒服的感受。拿到會議手冊後，一般就是需要好好規劃一番，想看看有沒有可合作的方向。

一路聽下來發覺到一些能夠繼續拿來研究的題材，收獲良多。在報告完本人的研究後發現，與會的聽眾都很專注地聆聽報告者的報告，並且從不同的角度提出的建議與肯定，這些意見確實使本人受到相當的啟發，受益良多。

四、建議

這次發表論文有許多是研究生，看到準備充分，演說自然，深深感到一定要持續努力，否則一定會被超越。非常感謝國科會與自然處提供此機會讓本人能夠出國參加國際研討會與國際研究接觸，對於個人未來的研究視野有著一定的正向影響作用，也提昇研究動機與能力。建議能夠鼓勵並補助更多同仁參與類似的會議，提升國際觀。

Applying the mountain pass theorem for a class of asymptotically linear Schrodinger equation

Kuan-Ju Chen

Abstract－In this paper, we consider the following nonlinear Schrodinger problem

$$\begin{cases} -\Delta u + (\lambda V(x) + Z(x))u = f(u) \text{ in } \mathbb{R}^N, \\ u \in H^1(\mathbb{R}^N),\ u > 0 \text{ in } \mathbb{R}^N, N \geq 3, \end{cases}$$

where $\lambda > 0$, $f(t)$ is asymptotically linear at infinity, that is, $f(t) \sim O(t)$ as $t \to \infty$. We don't require the following technical condition of the Ambrosetti-Rabinowtiz type, that is, for some $\theta > 2$,

$$0 \leq F(u) = \int_0^u f(t)dt \leq \frac{1}{\theta} f(u)u$$

for all $(x,u) \in \mathbb{R}^N \times \mathbb{R}$.

We use the so-called mountain pass geometry to prove that if V, Z and f satisfy some suitable conditions, there exists $\lambda^* > 0$, such that the problem admits at least one positive solution for $\lambda \in (0, \lambda^*)$.

Keywords－the mountain pass theorem, Schrodinger equation.

1. INTRODUCTION

We consider the existence of positive solutions for the following Schrodinger equation

$$\begin{cases} -\Delta u + (\lambda V(x) + Z(x))u = f(u) \text{ in } \mathbb{R}^N, \\ u \in H^1(\mathbb{R}^N),\ u > 0 \text{ in } \mathbb{R}^N, N \geq 3, (1.1) \end{cases}$$

where $\lambda \in (0, \infty)$ and we assume on the potentials $V(x)$, $Z(x) \in C(\mathbb{R}^N, \mathbb{R})$

(H1) there exists $M > 0$ such that $M \geq V(x) \geq 0$ for all $x \in \mathbb{R}^N$ and the potential well $\Omega := Int\ V^{-1}(0)$ is a non-empty bounded open set composed of k open connected components denoted by Ω_j, $j \in \{1, \ldots, k\}$, which satisfy $d(\Omega_i, \Omega_j) > 0$ for $i \neq j$, that is,

$$\Omega = \Omega_1 \cup \Omega_2 \cup \cdots \cup \Omega_k,$$

with smooth boundary $\partial\Omega$ and $V^{-1}(0) = \overline{\Omega}$;

(H2) there exist two positive constants M_0 and M_1 such that V and Z verify

$$0 < M_0 \leq \lambda V(x) + Z(x)$$
$$\text{for all } x \in \mathbb{R}^N \text{ and } \lambda > 0$$

and

$$1 \leq Z(x) \text{ for all } x \in \mathbb{R}^N.$$

and on the function $f(t) \in C^1(\mathbb{R}, \mathbb{R})$

(f1) $f(t) \geq 0$ if $t \geq 0$, and $f(t) = o(t)$ as $t \to 0$;

(f2) there is a constant $0 < l < +\infty$ such that $\lim_{t\to\infty} \frac{f(t)}{t} = l$ and $l > \inf \sigma(-\Delta + Z(x))$, where $\sigma(-\Delta + Z(x))$ denotes the spectrum of the self-adjoint operator $-\Delta + Z(x) : H^2(\mathbb{R}^N) \to L^2(\mathbb{R}^N)$;

(f3) $\sup_{t>0} \frac{f(t)}{t} < M_0$;

The main features of problem (1.1) is that the nonlinearity is asymptotically linear. Our main results are the following:

Theorem 1.1 Assume that (H1)-(H2), (f1)-(f3) hold, there exists $\lambda^* > 0$ such that problem (1.1) has at least one positive solution for all $\lambda \in (0, \lambda^*)$.

Theorem 1.2 Assume that (H1)-(H2), (f1)-(f3) hold. Then, for any non-empty subset Γ of $\{1, 2, \dots, k\}$, there exists λ^* such that, for $\lambda \geq \lambda^*$, problem $(P)_\lambda$ has a positive solution u_λ. Moreover, the family $\{u_\lambda\}_{\lambda \geq \lambda^*}$ has the following properties: For any sequence $\lambda_n \to \infty$, we can extract a subsequence λ_{n_i} such that u_{n_i} converges strongly in $H^1(\mathbb{R}^N)$ to a function u which satisfies $u(x) = 0$ for $x \notin \Omega_\Gamma$, and the restriction $u|_{\Omega_j}$ is a least energy solution of

$$-\triangle u + Z(x)u = f(u), \ u > 0 \quad \text{in } \Omega_j,$$
$$u|_{\partial\Omega_j} = 0 \quad \text{for} \quad j \in \Gamma,$$

where $\Omega_\Gamma = \cup_{j\in\Gamma} \Omega_j$.

As a corollary of Theorem 1.2, we have the following

Corollary 1.3 Under the assumptions of Theorem 1.2, there exists $\lambda^* > 0$ such that, for $\lambda \geq \lambda^*$, problem $(P)_\lambda$ has at least $2^k - 1$ positive solutions.

In all the above mentioned papers, the following technical condition of the Ambrosetti-Rabinowtiz type were imposed, that is, for some $\theta > 2$,

$$0 \leq F(u) = \int_0^u f(t)dt \leq \frac{1}{\theta} f(u)u \qquad (1.2)$$
$$\text{for all } (x, u) \in \mathbb{R}^N \times \mathbb{R}.$$

It is well known that the main role of (1.2) is to ensure the boundedness of all (PS)$_c$-sequences or minimizing sequence of the corresponding functional. By a simple calculation, (1.2) shows that $f(t)$ must be superlinear with respect to t at infinity, that is,

$$\lim_{t\to\infty} \frac{f(t)}{t} = \infty.$$

However, the study of many practical problems, e.g. special solutions of Maxwell's equations under some suitable constitutive assumptions, leads to some problems related to problem (1.1), in which $f(t)$ is asymptotically linear with respect to t at infinity. Without (1.2), it becomes more complicated.

2. PRELIMINARY REMARKS

In this section, we fix some notations and define some functionals used in this work.

Since we intend to find positive solutions, let us assume that

$$f(t) = 0 \ \text{ for all } t \in (-\infty, 0].$$

The nonnegative weak solutions of problem (1.1) are critical points of the functional $I_\lambda : H_\lambda \to \mathbb{R}$ given by

$$I_\lambda(u)$$
$$= \frac{1}{2}\int_{\mathbb{R}^N} |\nabla u|^2 + (\lambda V(x) + Z(x))u^2 - \int_{\mathbb{R}^N} F(u),$$

where $F(t) = \int_0^t f(\tau)$ and H_λ is the space of functions defined by

$$H_\lambda = \{u \in H^1(\mathbb{R}^N) : \int_{\mathbb{R}^N} (\lambda V(x) + Z(x))u^2 < \infty\}.$$

Here after, $\int_{\mathbb{R}^N} h$ denotes the integral $\int_{\mathbb{R}^N} hdx$. We define

$$\| u \|_\lambda = (\int_{\mathbb{R}^N} |\nabla u|^2 + (\lambda V(x) + Z(x))u^2)^{\frac{1}{2}}$$
$$\text{for } u \in H_\lambda$$

and we can easily see that $(H_\lambda, \| \cdot \|_\lambda)$ is a Hilbert space for $\lambda > 0$.

We also write for an open set $D \subset \mathbb{R}^N$

$$H_\lambda(D) = \{u \in H^1(D) : \int_D (\lambda V(x) + Z(x))u^2 < \infty\},$$

$$\| u \|_{\lambda,D} = (\int_D |\nabla u|^2 + (\lambda V(x) + Z(x))u^2)^{\frac{1}{2}}$$
$$\text{for } u \in H_\lambda(D).$$

We also define the following notations:

$$|u|_p^p = \int_{\mathbb{R}^N} |u|^p \, dx \ \text{ for } p \in [1, \infty),$$

$$\| u \|_{H^1(\mathbb{R}^N)}^2 = \int_{\mathbb{R}^N} (|\nabla u|^2 + M_0 u^2) dx.$$

Here $M_0 > 0$ is the constant appearing in (H2) and thus $\| \cdot \|_{H^1(\mathbb{R}^N)}$ is equivalent to the standard $H^1(\mathbb{R}^N)$-norm. Since

$$\| u \|_{H^1(\mathbb{R}^N)} \le \| u \|_\lambda, \quad (2.1)$$

we have $H_\lambda \subset H^1(\mathbb{R}^N)$ and H_λ can be embedded into $L^p(\mathbb{R}^N)$ ($2 \le p \le \frac{2N}{N-2}$ for $N \ge 3$, $2 \le p < \infty$ for $N = 2$) continuously, that is, there exists $C_p > 0$ such that

$$|u|_p \le C_p \| u \|_{H^1(\mathbb{R}^N)} \ \text{ for all } u. \quad (2.2)$$

In view of (H2),

$$M_0 \| u \|_{L^2(D)}^2 \le \| u \|_{\lambda,D}^2$$
$$\text{for all } u \in H(D) \text{ and } \lambda > 0.$$

We remark that when D is bounded, $H(D) = H^1(D)$ and $\| \cdot \|_{\lambda,D}$ is equivalent to $\| \cdot \|_{H^1(D)}$.

For each $j \in \{1, 2, \ldots, k\}$, we fix a bounded open subset Ω_j' with smooth boundary such that

(i) $\overline{\Omega_j} \subset \Omega_j'$,

(ii) $\overline{\Omega_j'} \cap \overline{\Omega_j'} = \phi$, for all $j \ne i$,

and for $\Gamma \subset \{1, 2, \quad, k\}$, $\Gamma \ne \phi$, let us fix

$$\Omega_\Gamma = \cup_{j \in \Gamma} \Omega_j \text{ and } \Omega_\Gamma' = \cup_{j \in \Gamma} \Omega_j'.$$

In what follows, c_j is the minimax level of Mountain Pass Theorem related to the functional $I_j : H_0^1(\Omega_j) \to \mathbb{R}$ given by

$$I_j(u)=\frac{1}{2}\int_{\Omega_j}|\nabla u|^2+Z(x)u^2-\int_{\Omega_j}F(u).$$

We know that the critical points of I_j are weak solutions of the following problem

$$\begin{cases}-\Delta u+Z(x)u=f(u) \ \text{ in } \Omega_j,\\ \qquad u=0 \ \text{ on } \partial\Omega_j.\end{cases}$$

3. AN AUXILIARY PROBLEM

In this section, we introduce a modification of $f(u)$ as in del Pino-Felmer [6] to obtain a family of solutions described in Theorem 1.1. Since we seek positive solutions, we can assume that $f(t)=0$ for all $t\le 0$.

Let $f(t)$ be a function satisfying (f1)-(f2). We choose a small number $\nu\in(0,\frac{M_0}{2})$ and we set $\widetilde{f}$, $\widetilde{F}:\mathbb{R}\to\mathbb{R}$ the following functions

$$\widetilde{f}(t)=\begin{cases}\min\{f(t),\nu t\} & \text{for } t\ge 0,\\ 0 & \text{for } t<0,\end{cases}$$

and $\widetilde{F}(t)=\int_0^t\widetilde{f}(\tau).$ By (f1) we can see that there exists a small $r_\nu>0$ such that

$$\widetilde{f}(t)=f(t) \ \text{ for } |t|\le r_\nu.$$

Moreover there holds

$$\widetilde{f}(t)=\nu t \ \text{ for large } t\ge 0,$$
$$\widetilde{f}(t)=0 \ \text{ for } t\le 0.$$

Note that

$$\widetilde{f}(t)\le f(t) \ \text{ for any } t\in\mathbb{R}. \quad (3.1)$$

In what follows, we fix non-empty subset $\Gamma\subset\{1,2,\dots,k\}$ and we try to find a positive solution described in Theorem

We set

$$\Omega_\Gamma=\cup_{j\in\Gamma}\Omega_j,\ \Omega'_\Gamma=\cup_{j\in\Gamma}\Omega'_j,$$
$$\chi_\Gamma(x)=\begin{cases}1, & \text{for } x\in\Omega'_\Gamma,\\ 0, & \text{for } x\notin\Omega'_\Gamma,\end{cases}$$

and let

$$g(x,t)=\chi_\Gamma(x)f(t)+(1-\chi_\Gamma(x))\widetilde{f}(t), \quad (3.2)$$
$$\text{for } (x,t)\in\mathbb{R}^N\times\mathbb{R},$$

$$G(x,t)=\int_0^t g(x,s)$$
$$=\chi_\Gamma(x)F(t)+(1-\chi_\Gamma(x))\widetilde{F}(t).$$

Moreover, under the conditions (H1)-(H2) and (f1)-(f3), the functional $\Phi_\lambda(u):H_\lambda\to\mathbb{R}$ given by

$$\Phi_\lambda(u)=\frac{1}{2}\int_{IR^N}|\nabla u|^2+(\lambda V(x)+Z(x))u^2$$
$$-\int_{IR^N}G(x,u) \qquad (3.3)$$

belongs to $C^1(H_\lambda,\mathbb{R})$ and its critical points are nonnegative weak solutions of

$$-\Delta u+(\lambda V(x)+Z(x))u=g(x,u)\quad \text{in } \mathbb{R}^N. \tag{3.4}$$

From now on we try to find a solution of the problem (3.4). We will find a solution $u_\lambda(x)$ of problem (1.1) via a version of the mountain pass theorem used in [5] and besides other properties we will show that the mountain pass solution $u_\lambda(x)$ satisfies $|u_\lambda(x)|\le r_\nu$ in $\mathbb{R}^N\setminus\Omega'_\Gamma$, that is, $u_\lambda(x)$ also solves the original problem.

We now recall the version of the mountain pass theorem.

Proposition 3.1 let E be a real Banach space with its dual space E^* and suppose that $I\in C^1(E,\mathbb{R})$ satisfies the condition

$$\max\{I(0),I(u_1)\}\le\alpha<\beta\le\inf_{\|u\|=\rho}I(u)$$

for some $\alpha<\beta$, $\rho>0$ and $u_1\in E$ with $\|u_1\|>\beta$. Let $c\ge\beta$ be characterized by

$$c=\inf_{\gamma\in\Theta}\max_{0\le t\le 1}I(\gamma(t))$$

where $\Theta=\{\gamma\in C([0,1],E):\gamma(0)=0,\gamma(1)=u_1\}$ is the set of continuous paths joining 0 and u_1. Then there exists a sequence $\{u_n\}\subset E$ such that

$$I(u_n)\xrightarrow{n}c\ge\beta$$

and

$$(1+\|u_n\|)\|I'(u_n)\|_{E^*}\xrightarrow{n}0.$$

For the functional defined by (3.3), we have the following lemma.

Lemma 3.2 Assume that (H1)-(H2) and (f1)-(f2) hold. Then we have

(1) there exist $\rho_0>0$ and $\beta>0$ independent of $\lambda>0$ such that

$$\Phi_\lambda(u)\ge\beta>0 \text{ for all } \|u\|_\lambda=\rho_0,$$
$$\Phi_\lambda(u)>0 \text{ for all } 0<\|u\|_\lambda\le\rho_0.$$

(2) let $\lambda<\frac{-\sigma}{M}$ where $\sigma=\inf\{\int_{\mathbb{R}^N}(|\nabla u|^2+Z(x)u^2-lu^2)dx: u\in H^1(\mathbb{R}^N)\text{ with }|u|_2^2=1\}$, then there exists $e\in H^1(\mathbb{R}^N)$ such that $\|e\|_\lambda>\rho_0$ and $\Phi_\lambda(e)<0$.

Proof. (1) Given $\varepsilon>0$, $p\in(1,(N+2)/(N-2))$, by (f1) and (f2), there exists $C_\varepsilon>0$ such that

$$0\le f(t)\le\varepsilon|t|+C_\varepsilon|t|^p,\quad \text{for all } t\in\mathbb{R}, \tag{3.5}$$

$$0\le F(t)\le\varepsilon|t|^2+C_\varepsilon|t|^{p+1},\quad \text{for all } t\in\mathbb{R}. \tag{3.6}$$

Taking $\varepsilon=M_0/4$ in (3.6), using Sobolev embedding and Hölder's inequality, we see that, for any $u\in H_\lambda(\mathbb{R}^N)$,

$$\Phi_\lambda(u)=\frac{1}{2}\|u\|_\lambda^2$$
$$-\int_{\mathbb{R}^N}(\chi_\Gamma(x)F(u)+(1-\chi_\Gamma(x))\widetilde{F}(u))dx$$
$$\ge\frac{1}{2}\|u\|_\lambda^2-\int_{\mathbb{R}^N}F(u)dx$$
$$\ge\frac{1}{2}\|u\|_\lambda^2-\frac{1}{4}\int_{\mathbb{R}^N}M_0|u|^2\,dx$$
$$-C_\varepsilon\int_{\mathbb{R}^N}|u|^{p+1}\,dx$$
$$\ge\frac{1}{2}\|u\|_\lambda^2-\frac{1}{4}\|u\|_\lambda^2-C_\varepsilon|u|_{p+1}^{p+1}$$
$$\ge(\frac{1}{4}\|u\|_\lambda-C_\varepsilon C_{p+1}^{p+1}\|u\|_\lambda^p)\|u\|_\lambda.$$

Noting a function $S(\rho)=\frac{1}{4}\rho-C_\varepsilon C_{p+1}^{p+1}\rho^p$ satisfies that $S(0)=0$, $S'(0)>0$, $S(+\infty)=-\infty$ and we find a $\rho_0>0$ such that $S(\rho_0)=\sup_{\rho\ge0}S(\rho)>0$. Setting $\beta=S(\rho_0)\rho_0$, we have $\Phi_\lambda(u)\ge\beta>0$ for all $\|u\|_\lambda=\rho_0$.

(2) Since $\lambda<\frac{-\sigma}{M}$, there exists an $\varepsilon>0$ such that $\lambda+\frac{\sigma}{M}+\frac{\varepsilon}{M}<0$. For this $\varepsilon>0$, from the definition of σ, we may choose $0\le\phi\in H^1(\mathbb{R}^N)$ such that

$$|\phi|_2^2=1$$
$$\text{with}\int_{\mathbb{R}^N}(|\nabla\phi|^2+Z(x)\phi^2-l\phi^2)dx<\sigma+\varepsilon.$$

Therefore, by (f2) and the Fatou's Lemma we deduce that, for $t>0$,

$$\lim_{t\to\infty}\frac{\Phi_\lambda(t\phi)}{t^2}=\frac{1}{2}\|\phi\|_\lambda^2-\lim_{t\to\infty}\int_{\mathbb{R}^N}\frac{G(x,t\phi)}{t^2}dx$$
$$\le\frac{1}{2}\int_{\mathbb{R}^N}(|\nabla\phi|^2+Z(x)\phi^2-l\phi^2)dx$$
$$+\frac{1}{2}\lambda\int_{\mathbb{R}^N}^2V(x)\phi^2dx$$
$$\le\frac{1}{2}(\lambda M+\sigma+\varepsilon)<0,$$

then for a sufficiently large $t_0>0$ such that $e=t_0\phi$ with $\|e\|_\lambda>\rho_0$ and $\Phi_\lambda(e)<0$.

We look for a positive solution of problem (1.1) by mountain pass type argument used in [5]. By Lemma 3.2, we can define the mountain pass value. For $\lambda\in(0,\frac{-\sigma}{M})$, we set

$$c_\lambda=\inf_{\gamma\in\Gamma_\lambda}\max_{t\in[0,1]}\Phi_\lambda(\gamma(t)),$$

where

$$\Gamma_\lambda=\{\gamma(t)\in C\left([0,1],H_\lambda(\mathbb{R}^N)\right):$$
$$\gamma(0)=0,\quad\gamma(1)=e\},$$

where e is given in Lemma 3.2. From Lemma 3.2 and Ekeland's principle, for any $\lambda\in(0,\frac{-\sigma}{M})$, the functional Φ_λ has a MP geometry, we deduce (see [5]) the existence of a Cerami sequence at the MP level c_λ, namely of a $\{u_n\}\subset H_\lambda$ such that

$\Phi_\lambda(u_n) \to c_\lambda$

$(1+\|u_n\|_\lambda)\|\Phi'_\lambda(u_n)\|_{H_\lambda^{-1}(\mathbb{R}^N)} \to 0$ (3.7)

as $n \to \infty$,

where $H_\lambda^{-1}(\mathbb{R}^N)$ denotes the dual space of $H_\lambda(\mathbb{R}^N)$. At this point to get an existence result it clearly suffices to show that $\{u_n\}$ is bounded and then that $\{u_n\}$ has a convergent subsequence whose limit is a nontrivial critical point of Φ_λ. Thus $\Phi_\lambda(u)$ has a critical point u_λ satisfying $\Phi'_\lambda(u_\lambda)=0$ and $\Phi_\lambda(u_\lambda)=c_\lambda$. Also we show that $\{u_n\}$ is bounded.

By the above Lemma 3.2, we have the following a priori bound for the mountain pass value c_λ.

Corollary 3.3 There are constants $b_1>0$, $b_2>0$ such that for $\lambda \in (0, \frac{-\sigma}{M})$

$$b_1 \le c_\lambda \le b_2. \quad (3.8)$$

Proof. By Lemma 3.2, we have

$$\max_{t\in[0,1]} \Phi_\lambda(\gamma(t)) \ge \inf_{\|u\|_\lambda=\rho_0} \Phi_\lambda(u) \ge \beta.$$

On the other hand, taking a path $\gamma_0(t)=te$, where e is given in Lemma 3.2, we have

$$c_\lambda \le \sup_{\lambda\in(0,\frac{-\sigma}{M})} \max_{t\in[0,1]} \Phi_\lambda(\gamma_0(t)) \equiv b_2.$$

Thus we get (3.8) with $b_1=\beta$ and b_2 given in the above formula.

Now we will discuss the boundedness of Cerami sequences corresponding to c_λ and has a convergent subsequence.

Lemma 3.4 Assume that (H1)-(H2) and (f1)-(f3) hold. For any $\lambda \in (0, \frac{-\sigma}{M})$, if $\{u_n\} \subset H_\lambda(\mathbb{R}^N)$ is a Cerami sequence for Φ_λ, then we have

(i) $\{u_n\}$ is bounded.

(ii) There exist a subsequence n_k and $u_0 \in H_\lambda$ such that $u_{n_k} \to u_0$ strongly in H_λ.

Without the condition of the Ambrosetti-Rabinowtiz type (1.2), it is difficult to show that a Cerami sequence is bound. Now we establish some preliminary results to prove the Cerami sequence $\{u_n\}$ is bounded in $H_\lambda(\mathbb{R}^N)$. Let $w_n = u_n/\|u_n\|_\lambda$. Clearly, w_n is bounded in $H_\lambda(\mathbb{R}^N)$ and there exists $w \in H_\lambda(\mathbb{R}^N)$ such that, up to a sequence, as $n\to\infty$,

$$\begin{cases} w_n \rightharpoonup w \text{ weakly in } H_\lambda(\mathbb{R}^N), \\ w_n \to w \text{ a.e. in } \mathbb{R}^N, \\ w_n \to w \text{ strongly in } L^2_{loc}(\mathbb{R}^N). \end{cases} \quad (3.9)$$

Lemma 3.5 Assume that (H1)-(H2) and (f1)-(f3) hold. If $\|u_n\|_\lambda \to +\infty$ as $n\to\infty$, then w given by (3.9) is a nontrivial nonnegative solution of

$$\begin{aligned} &-\Delta u + (\lambda V(x) + Z(x))u \\ &= (\chi_\Gamma(x) l + (1-\chi_\Gamma(x))\nu)u, \quad (3.10) \\ &u \in H_\lambda(\mathbb{R}^N). \end{aligned}$$

Proof. We prove this lemma through the following three steps.

Step1. $w \not\equiv 0$.

Since $\Omega := Int \ V^{-1}(0)$ is a non-empty bounded open set, there exist $\overline{R}>0$ such that $\Omega \subset B_R(0)$ and then by (3.2), (3.1) and (f3), we have

$$\sup_{|x|\geq \bar{R}, t>0} \frac{g(x,t)}{t} = \sup_{|x|\geq \bar{R}, t>0} \frac{\tilde{f}(t)}{t} \leq \sup_{t>0} \frac{f(t)}{t} < M_0,$$

here and below $B_R(0) = \{x \in \mathbb{R}^N : |x| < R\}$, then for all $n \in \mathbb{N}$,

$$\begin{aligned} &\int_{|x|\geq \bar{R}} \frac{g(x,u_n)}{u_n} w_n^2 dx \\ &< M_0 \int_{|x|\geq \bar{R}} w_n^2 dx \qquad (3.11) \\ &\leq \int_{|x|\geq \bar{R}} (\lambda V(x) + Z(x)) w_n^2 dx < 1. \end{aligned}$$

By contradiction, if $w \equiv 0$, since the embedding $H_\lambda(B_R(0)) L^2(B_R(0))$ is compact, $w_n \to 0$ strongly in $L^2_{loc}(IR^N)$ as $n \to \infty$ and, by (f1) and (f2), there exists $\dot{C} > 0$ such that

$$\frac{g(x,t)}{t} \leq \frac{f(t)}{t} \leq C, \quad \text{for all } t \in \mathbb{R}, \qquad (3.12)$$

hence,

$$\begin{aligned} &\int_{|x|<\bar{R}} \frac{g(x,u_n)}{u_n} w_n^2 dx \\ &\leq C \int_{|x|<\bar{R}} w_n^2 dx \to 0 \text{ as } n \to \infty. \end{aligned} \qquad (3.13)$$

Therefore, (3.11) and (3.13) give that

$$\limsup_{n\to\infty} \int_{\mathbb{R}^N} \frac{g(x,u_n)}{u_n} w_n^2 dx < 1. \qquad (3.14)$$

However, since $\|u_n\|_\lambda \to +\infty$ as $n \to \infty$, it follows from (3.7) that

$$\frac{<\Phi_\lambda'(u_n), u_n>}{\|u_n\|_\lambda^2} = o(1),$$

that is,

$$o(1) = 1 - \int_{IR^N} \frac{g(x,u_n)}{u_n} w_n^2 dx,$$

where, and in what follows, $o(1)$ denotes a quantity which goes to zero as $n \to \infty$. Clearly, this contradicts (3.14). So, $w \not\equiv 0$.

Step 2. $w \geq 0$.

Let $w_n^-(x) = \max\{-w_n(x), 0\}$, $w_n^-(x)$ is also bounded in $H_\lambda(\mathbb{R}^N)$. If $\|u_n\|_\lambda \to +\infty$, then

$$\frac{<\Phi_\lambda'(u_n), w_n^->}{\|u_n\|_\lambda} = o(1),$$

that is,

$$\begin{aligned} &-\|w_n^-\|_\lambda^2 \\ &= \int_{IR^N} \frac{g(x, \|u_n\|_\lambda w_n)}{\|u_n\|_\lambda} w_n^- dx + o(1). \end{aligned} \qquad (3.15)$$

Since $g(x,t) \equiv 0$ if $t \leq 0$, it follows from (3.15) that $\|w_n^-\|_\lambda = o(1)$. Thus $w^- = 0$ a.e. in $x \in \mathbb{R}^N$ and $w \geq 0$.

Step 3. w solves (3.10).

By (3.7) and $\|u_n\|_\lambda \to +\infty$ as $n \to \infty$, we have

$$\frac{<\Phi'_\lambda(u_n),\phi>}{\|u_n\|_\lambda}=o(1),\quad \text{for any } \phi\in C_0^\infty(\mathbb{R}^N),$$ that is,

$$\int_{\mathbb{R}^N}(\nabla w_n\nabla\phi+(\lambda V(x)+Z(x))w_n\phi)dx$$
$$=\int_{\mathbb{R}^N}\frac{g(x,u_n)}{u_n}w_n\phi dx+o(1).$$

Since $w_n \rightharpoonup w$ weakly in $H^1(\mathbb{R}^N)$ as $n\to\infty$, we see that

$$\int_{\mathbb{R}^N}(\nabla w\nabla\phi+(\lambda V(x)+Z(x))w\phi)dx$$
$$=\int_{\mathbb{R}^N}\frac{g(x,u_n)}{u_n}w_n\phi dx+o(1).$$

So, to show w solves (3.10) we only need have that

$$\int_{\mathbb{R}^N}\frac{g(x,u_n)}{u_n}w_n\phi dx$$
$$\to\int_{\mathbb{R}^N}(\chi_\Gamma(x)l+(1-\chi_\Gamma(x))\nu)w\phi dx \quad (3.16)$$
$$\text{as } n\to\infty.$$

In fact, by (3.12) and (H2),

$$\int_{\mathbb{R}^N}\left|\frac{g(x,u_n)}{u_n}w_n\right|^2dx$$
$$\le C\int_{\mathbb{R}^N}w_n^2dx \qquad (3.17)$$
$$\le C\int_{\mathbb{R}^N}Z(x)w_n^2dx\le C,$$

that is, $\{\frac{g(x,u_n)}{u_n}w_n\}$ is bounded in $L^2(\mathbb{R}^N)$. Let

$$\Lambda_+=\{x\in\mathbb{R}^N : w(x)>0\}$$

and

$$\Lambda_0=\{x\in\mathbb{R}^N : w(x)=0\}.$$

Noting that

$$w_n(x)=\frac{u_n(x)}{\|u_n\|_\lambda}\to w(x)\ \text{ a.e. in } \mathbb{R}^N$$

and

$$\|u_n\|_\lambda\to+\infty \text{ as } n\to\infty$$

then $u_n(x)\to\infty$ a.e. in $x\in\Lambda_+$ as $n\to\infty$. Hence (f2) implies that

$$\frac{g(x,u_n)}{u_n}w_n(x)$$
$$\to(\chi_\Gamma(x)l+(1-\chi_\Gamma(x))\nu)w(x)\ \text{ a.e.}$$
$$\text{in } x\in\Lambda_+ \ \text{ as } n\to\infty.$$

Since $w_n(x)\to 0$ a.e. in $x\in\Lambda_0$ as $n\to\infty$, it follows from (3.12) that

$$\frac{g(x,u_n)}{u_n}w_n(x)$$
$$\to 0 \equiv (\chi_\Gamma(x)l + (1-\chi_\Gamma(x))\nu)w(x)$$
$$\text{a.e. in } x \in \Lambda_0 \ \text{ as } n \to \infty.$$

These and (3.17) imply that

$$\frac{g(x,u_n)}{u_n}w_n(x)$$
$$\rightharpoonup (\chi_\Gamma(x)l + (1-\chi_\Gamma(x))\nu)w(x) \quad (3.18)$$
$$\text{weakly in } L^2(\mathbb{R}^N) \ \text{ as } n \to \infty.$$

By $\phi \in C_0^\infty(\mathbb{R}^N)$, we know that $\phi \in L^2(\mathbb{R}^N)$, and then (3.18) implies (3.16).

Lemma 3.6 Assume that (H1)-(H2) and (f1)-(f3) hold. If $\lambda < \frac{-\sigma}{M}$ where $\sigma = \inf\{\int_{IR^N}(|\nabla u|^2 + Z(x)u^2 - lu^2)dx : u \in H^1(IR^N) \text{ with } |u|_2^2 = 1\}$, then problem (3.10) has no nontrivial nonnegative solutions.

Proof. Seeking a contradiction, suppose that $u \in H^1(\mathbb{R}^N)$ is a nontrivial nonnegative solution of problem (3.10). First, by the definition of σ, there exists $v \in H^1(\mathbb{R}^N)$ satisfying

$$\int_{\mathbb{R}^N}(|\nabla v|^2 + Z(x)v^2 - lv^2)dx$$
$$< -\lambda M \int_{\mathbb{R}^N} v^2 dx.$$

Thus, since $C_0^\infty(\mathbb{R}^N)$ is dense in $H^1(\mathbb{R}^N)$, we may assume $v \in C_0^\infty(\mathbb{R}^N)$. Now, let $R > 0$ be such that $\Omega_\Gamma \subset B_R(0)$ and $\mathrm{supp} v \subset B_R(0)$ and consider the Dirichlet problem for $-\Delta + (Z(x) - l)$ on $B_R(0)$. Denote by μ_R the infimum of the spectrum of $-\Delta + (Z(x) - l)$ on $B_R(0)$. By construction,

$$\mu_R$$
$$\le \frac{\int_{\mathbb{R}^N}(|\nabla v|^2 + Z(x)v^2 - lv^2)dx}{|v|_2^2} \quad (3.19)$$
$$< -\lambda M.$$

On the other hand, μ_R is an eigenvalue of $-\Delta + (Z(x) - l)$ associated to an eigenvector $v_R \ge 0$ on B_R. It follows from the strong maximum principle that

$$v_R(x) > 0 \ \text{ for all } x \in B_R(0),$$
$$\frac{\partial v_R(x)}{\partial n} < 0, \ \text{ for all } |x| = R.$$

Therefore, if $0 \not\equiv u \in H^1(\mathbb{R}^N)$ is a nonnegative solution of problem (3.10), then

$$\begin{aligned}
&\mu_R < u, v_R >_{B_R(0)} \\
&=< u, (-\Delta + Z(x) - l)v_R >_{B_R(0)} \\
&= \int_{B_R(0)} \nabla u \cdot \nabla v_R + \int_{B_R(0)} (Z(x) - l) u v_R \\
&- \int_{\partial B_R(0)} \frac{\partial v_R(x)}{\partial n} u d\sigma \\
&\geq \int_{B_R(0)} (-\Delta u + (Z(x) - l)u) v_R \\
&=< (-\Delta + Z(x) - l)u, v_R >_{B_R(0)} \\
&= -\lambda \int_{B_R(0)} V(x) u v_R \\
&\geq -\lambda M \int_{B_R(0)} u v_R \\
&= -\lambda M < u, v_R >_{B_R(0)},
\end{aligned}$$

where $< \cdot, \cdot >_{B_R(0)}$ denotes the scalar product of $L^2(B_R(0))$. But since $u \geq 0$ and $v_R \geq 0$, we may choose $R > 0$ large enough such that $< u, v_R >_{B_R(0)} > 0$. So, the above calculation shows that $\mu_R \geq -\lambda M$ in contradiction with (3.19).

Now, we give the proof of the Lemma 3.4.

Proof of the Lemma 3.4. Clearly, if $\| u_n \|_\lambda \to +\infty$ as $n \to \infty$, from Lemma 3.5 and Lemma 6, we get a contradiction. Hence, $\{u_n\}$ is bounded in $H_\lambda(\mathbb{R}^N)$.

To prove (ii) of Lemma , we fix $\lambda \in (0, \frac{-\sigma}{M}]$ and $\{u_n\}$ satisfying (3.7). After extracting a subsequence if necessary, we may assume that $u_n \rightharpoonup u_0$ weakly in H_λ. It is sufficient to prove that for any $\varepsilon > 0$, there exist $R(\varepsilon) > R_0$ (R_0 is given by $\Omega_\Gamma' \subset B_{\frac{R_0}{2}}(0)$) and $n(\varepsilon) > 0$ such that

$$\int_{|x| \geq R} |\nabla u_n|^2 + (\lambda V(x) + Z(x)) u_n^2 \leq \varepsilon, \quad \text{for all } R \geq R(\varepsilon) \text{ and } n \geq n(\varepsilon). \tag{3.20}$$

Let $\eta_R : \mathbb{R}^N \to [0,1]$ be a smooth function such that

$$\eta_R(x) = \begin{cases} 0, & 0 \leq |x| \leq \frac{R}{2} \\ 1 & |x| \geq R. \end{cases} \tag{3.21}$$

Moreover, there exists a constant C_0 independent of R such that

$$|\nabla \eta_R(x)| \leq \frac{C_0}{R} \text{ for all } x \in \mathbb{R}^N. \tag{3.22}$$

Then for any $u \in H_\lambda(\mathbb{R}^N)$ and all $R \geq 1$, there exists a constant $C_1 > 0$ such that $\| \eta_R u \|_\lambda \leq C_1 \| u \|_\lambda$ Since $< \Phi_\lambda'(u_n), \eta_R u_n >= o(1)$, we know that, for any $\varepsilon > 0$, there exists $n(\varepsilon) > 0$ such that

$$\begin{aligned}
&< \Phi_\lambda'(u_n), \eta_R u_n > \\
&\leq C_1 \| \Phi_\lambda'(u_n) \|_{H_\lambda^{-1}(\mathbb{R}^N)} \| u_n \|_\lambda \\
&\leq \frac{\varepsilon}{4}, \text{ for } n \geq n(\varepsilon),
\end{aligned}$$

that is, if $n \geq n(\varepsilon)$, we have for sufficiently large $R(\varepsilon) > R_0$ with $\Omega_\Gamma' \subset B_{R_0}(0)$,

$$\int_{\mathbb{R}^N} (|\nabla u_n|^2 + (\lambda V(x) + Z(x))u_n^2)\eta_R$$

$$+u_n \nabla u_n \cdot \nabla \eta_R$$

$$\leq \int_{\mathbb{R}^N} \widetilde{f}(u_n)\eta_R u_n + \frac{\varepsilon}{4}$$

$$\leq \nu \int_{\mathbb{R}^N} \eta_R |u_n|^2 + \frac{\varepsilon}{4}.$$

Then for $R \geq R(\varepsilon)$ and $n \geq n(\varepsilon)$, combing (3.21), (3.22) and (3.22) we deduce that

$$\int_{IR^N} (|\nabla u_n|^2 + (\lambda V(x) + Z(x) - \nu)u_n^2)\eta_R$$

$$\leq \frac{C_0}{R} \| u_n \|_\lambda^2 + \frac{\varepsilon}{4} \leq \frac{C_2}{R} + \frac{\varepsilon}{4}.$$

Noting that the constant C_2 is independent of R, we can choose $R > 0$ large enough such that (3.20) holds.

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